INAUGURAL LECTURE

MATHEMATICS IN GREAT CULTURES

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Introduction

It is my great privilege to have opportunity to address this lecture to the University Authorities, to the Academic Staff, to Students and to distinguished Guests.

At the beginning, let me express my thanks to the Public Education Unit for the initiative to present this lecture and to the Library Staff for efficient and excellent service. In order to prepare this lecture, I have used the University Library to access a great collection of books, journals, the internet literature and comfortable space.

Under the broad title "Mathematics in Great Cultures", I would be able to present only selected historical facts and people whose names are well known in the History of Mathematics.

The Greek word Mathematics was probably used for the first time by Pythagoras in the fifth century B.C. In Greek, Mathematics means knowledge which is learned and understood. Aside possibly from Astronomy, Mathematics is the oldest and most continuously pursued branch of human thought. In the Ancient Greek philosophy, Mathematics was regarded as part of the existing Ideal World.

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1. Mathematics in Ancient Times

1.1. The Ancient Concept of Natural Numbers

Since deep past of human history, simple counting and arithmetic operations on natural numbers became imperative. The concept of numbers and the simple counting were known as far back as 50,000 years ago. This is evident from archaeological and historical records.
In the cultures of ancient Egypt and Greece, people believed that all knowledge about numbers, geometry, stars and movement of the Sun, the Moon and the planets has been conveyed to Priests by Gods.

Numbers were respected as living godly beings which had great influence on earthly life. These beliefs concerned natural numbers such as one, two three,...; Carl F. Gauss (1777-1855), universally regarded as one of the three greatest mathematicians of all time along with Archimedes (287 - 190, B.C.) and Isaac Newton (1642 - 1727 A.D.), expressed the opinion about natural numbers saying that "Natural Numbers have been created by God and any other numbers such as rational or irrational were developed by people". Numbers have been used in daily life by everybody and everywhere - by adults and children, rich and poor alike - and have never been exhausted or destroyed because of their external existence.

In our daily life and scientific activities, we use a small part of all existing numbers. Even in the developed techniques of computer calculations, a small subset of numbers with a finite number of digits is used. This small subset of computer numbers does not even create a number field, and it is much smaller than the smallest existing field of rational numbers. Within applications, operations on numbers are always done with a certain level of accuracy. This dictated the necessity of developing the theory of round-off error analysis as one of the important subjects within Computational Mathematics.

1.2. Mathematics in Babylon.

The oldest text from 2100 B.C. with notes in mathematics was found in Mesopotamia in the town of Ur. This text contains tables of additions and multiplications of natural numbers, at the base of 60. This system of numbers survived until contemporary times. Another clay table dated to 1950 B.C. was found in Babylon. In this table, there are four arithmetic operations, squares, cubics and square root of some natural numbers. Also, in that table, there is solution x=30 of the quadratic equation.

$$\frac{13}{9}x^2 - \frac{40}{3}x - 900 = 0$$

solved by the method of completing to squares.

Also, an amazing fact is that people in Babylon knew how to compute the square root $\sqrt{2}$ with great accuracy using the iterative formula

$$x_{n+1} = \frac{1}{2}(x_n + \frac{2}{x_n})$$

for n=0,1,2,...
This formula is a particular case of Newton's iterative method

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}, \text{ when } F(x) = x^2 - 2 = 0.$$

discovered about 3000 years later.

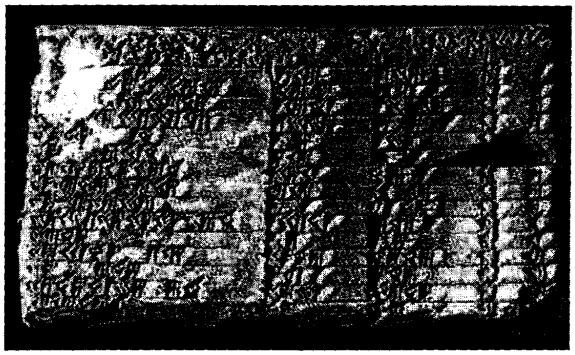
Also in Babylon, people used the formula

$$1^2+2^2+3^2+\ldots+n^2=\frac{1}{6}n(n+1)(2n+1)$$

to evaluate the volume of Babylon Tower. The rigorous proof of this formula was given over one thousand years later by Archimedes (287 - 212 B.C.)
One of the most remarkable discovery was the clay table Plimtom 322 dated from 1900 - 1600 B.C. In that table Pythagorean Triples were found , that is, three natural numbers a,b,c such that

$$a^2+b^2=c^2, \qquad 3^2+4^2=5^2$$

These relations satisfy three sides of a right triangle and it is known as Pythagoras Theorem rediscovered by Pythagoreans about 1000 years later in the Pythagorean School (569-500).



Plimton 322 Table

1.3. Mathematics in Egypt

In Egypt the earliest historical records on numbers, geometry and astronomy were found in the fourth millennium B.C., at the time when Egyptian Priests were able to determine the location of Sirius with respect to the position of Sun. They used this knowledge to create the first calendar in 4241 B.C., with a year consisting of 365 days. This oldest historical date from the fourth millennium B.C. was found in the writing of the Egyptian Priests.

In 2600 B.C., the Great Pyramid in Gizeh was built and without doubt, a lot of mathematics was done in that engineering construction.
On the Egyptian Papyrus dated from 1850 B.C., there is the formula

$$V=\frac{h}{3}(a^2+ab+b^2)$$

which represents the volume of a cone in the shape of a pyramid.

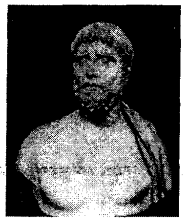
Papyrus Rhind (1850 B.C)., contains four arithmetic operations, square root and the solution of a linear equation. Ancient Egyptians wrote numbers in a system which is very similar to the decimal system.

1.4. Greek Mathematics

Since at least 6000 B.C., the land of Ancient Greece was inhabited by many migrating groups of people. There were Indo-Europeans, Achaeans, people from the Black and the Caspian Sea. The land of Ancient Greece reached as far as Turkey on the east, Italy on the west, Yugoslavia, Bulgaria and Romania on the north, all islands on the Ionian and the Adriatic Sea from the south. In the period of Alexander the Great Empire (350- -200 B.C.), Ancient Greece assimilated achievements of all cultures of the Mediterranean Sea. Many educated Greeks went through the merchant routes to Palestine, Egypt, Mesopotamia and India.

First, Greeks learned Astronomy, Architecture, Medicine, Mathematics and Physics from the other cultures. Then, they became the best teachers leaving behind well-documented literature on Mathematics and Science for the generations to come. As an important part of their teaching activities, the Ancient Greeks organized Schools of Philosophy, Mathematics and Science on the territory of Greece, Egypt and Mesopotamia.

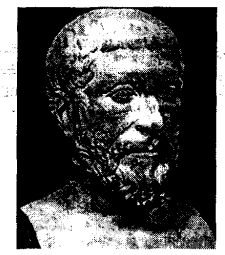
1.5. Thales (625-545 B.C.)



Ionian School. Ionian School of Astronomy, Mathematics and Philosophy was founded by Thales of Millets. It was the first school which had great influence on the entire civilized world. For a long period of time, Thales was in Egypt, where he learned Egyptian Astronomy, Geometry and Arithmetic.

Thales predicted the eclipse on the 28th of May, 585 B.C. in the period of wars among Medov tribes inhabiting the present territory of Iran and Turkey. He knew the periodicity of eclipse with the circle every 18 years and 11 days. Thales also knew that a year had 365 days, instead of exactly twelve months, 30 days each. In Geometry, there are well known Thales theorems on proportionality of intervals determined by a section of an angle, which have been taught in High School Programs since two thousand years ago. The Ionian School survived until about 400 B.C.

1.6. Pythagoras (569 - 500 B.C)



Pythagorean School. Pythagoras was born in a Greek family on the island of Samos located on Turkey's west coast. In the 6th century B.C., his parents probably immigrated from the town of Tyre (at present Haifa in Israel) to Greece.

Pythagoras was about 50 years younger then Thales, and he probably met him at some point. It is known from the records that he studied under Thales's disciples, Pherekydes and Anaksimander.

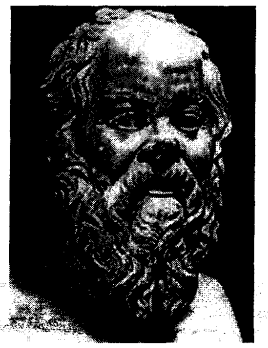
Pythagoras spent some time in Egypt and Babylon, where he studied Egyptian and Babylonian Mathematics and Philosophy. Possibly he was also in India where he learned about Hinduism. Around 530 B.C., Pythagoras settled in Croton, a Greek town in southern Italy. There, Pythagoras founded a mystical school on Philosophy and Mathematics to which all social classes of people where admitted, rich and poor, boys and girls alike. The School was characterized by great discipline. Pythagoreans were vegetarians eating simple meals. They considered meat as food for the beasts. Offering animals to gods was a crime against the dignity of life. Pythagoras even went to the extreme of saying that people will continue to kill themselves as long as they kill animals. Pythagoreans believed in the incarnation of souls.

In Mathematics, Pythagoreans regarded numbers as ideal beings transcendentally existing beyond time and space. The number one (1) was the most important among all integers, since by addition, all numbers come from one. The number ten (10) had a special privilege, since it formed the base in the decimal system. Also, the sum of four consecutive integers (1, 2, 3 and 4) was special, since it represented three dimensions, one (1) stood for the dimension of a point, two (2) the dimension of a line, three (3) the dimension of a plane and four (4) stood for the dimension of space. The relation between the Pythagorean triples of integers a,b,c such that

$$a^2 + b^2 = c^2$$

was first proved by Pythagoreans. The triples were known in Babylon about 1000 years earlier. At that time, the influence of the brotherhood and aristocratic tendencies became so great that the administrative forces of southern Italy destroyed the buildings of the school and dispersed the society of Pythagoreans. Pythagoras was probably murdered at the age of 75-80. The brotherhood scattered and still continued to exist for at least two centuries.

Socrates (469-399 B.C.)



In the period of 420 - 300 B.C., Athens became the center of Philosophy and Science. It was then that Socrates became the Father of Greek philosophy. He considered mathematics as the most important subject in all aspects of teaching. At 70 years old, Socrates was condemned and sentenced to death by his opponents for teaching about the democratic social system against Greek aristocracy. Socrates was a poor man, dressed in the same coat and the same sandals throughout his life. He was also a modest man saying "I know that I know nothing". In education, he was of the opinion that teaching should be free and not paid for as a common service. Socrates went down in history as the defender of social justice and human dignity.

Plato (429-348 B.C.) and Aristotle (384-322 B.C.)

1.8. Plato Academy.

One of Socrates' disciples was the great idealist Plato (429 - 348 A.D.) Plato was born in Athens in a rich aristocratic family. In the gardens which he inherited, Plato built his famous Academy of Philosophy. The Pythagorean School and the Plato's Academy of Philosophy paid great attention to Mathematics as the knowledge sent from heaven. This prominence was expressed by the following inscription at the door leading to the Academy "Let no one ignorant of mathematics enter here".
In his idealistic philosophy, Plato taught that the Universe consists of Matter and Spirit. The non-material Spiritual World is perfect and good, but the Material World is imperfect and wrong. Plato taught that "if we have true knowledge, we must leave the body. In the body, the soul cannot have true knowledge". The best way to see and to understand the Spiritual World is through Mathematics which alone is perfect.
Plato's disciples, among them Aristotle (384 - 322 B.C.), continued teaching Philosophy and Mathematics for many years to come in the School of Athens.
The idealistic philosophy taught later had great influence on the cultures over the World including Christian Culture.

1.9. University of Alexandria

In the third century B.C., when Alexander the Great conquered the ancient world, he built in Egypt the most cosmopolitan town called Alexandria. It was then and there that the first university with lecture rooms and great library were founded. Lecturers from different countries and cultures (Greeks and Jews, Babylonians, Syrians and Egyptians) were employed at the university. Amongst them, Euclid (330-275 B.C), Archimedes (287-212 B.C), and Apollonius (250- 190 B.C).
After the death of Alexander the Great (322 B.C), Ptolemy succeeded Egypt as part of the Greek Empire. Ptolemy was of great enthusiasm, he developed the University and built the library. The library contained 600,000 rolls with about 90,000 distinct works.

Euclid (330-275 B.C).

Ptolemy invited Euclid who opened the Mathematical School and was appointed as the Professor and the Dean of Science. For the first time at the University of Alexandria, Mathematics was separated from other sciences as one subject on Arithmetic and Geometry. Euclid went down in the history as one of the greatest mathematicians in the ancient world and the author of Elements, the book which has been used for thousands of years.

Archimedes (287-212 B.C).

Archimedes from Syracuse was the son of the astronomer, Phidias, and cousin of the King of Sicily, Heron. The King Heron is known in Mathematics as the author of the formula for calculation of the surface of a triangle. Heron is also known as the King who had not harmed or killed anybody in his Kingdom during his long reign of 54 years.

Plutarch wrote the following: During the siege of the city, Marcellus wished to preserve Archimedes' life, but he was slain by accident. In Archimedes honor, Marcellus raised a tomb bearing the figure of a sphere inscribed in a cylinder. Cicero had the honor of restoring the tomb during his stop in Sicily in 75 B.C. Archimedes' tomb was forgotten and lost for thousands of years. About 2000 years later in 1957, the tomb was found during a construction.

The scripture on the tomb says "Many are convinced that knowledge on nature and life on Earth is conveyed by messengers from heaven".
The University of Alexandria continued to exist under Ptolemy's rule for the next 350 years up to 30 A.D.

2. Mathematics in Middle Ages and Renaissance

The period from the fall of the Roman Empire in 476 A.D. to the fall of Constantinople in 1453 A.D. is known as Middle Ages. After the fall of Constantinople which was conquered by the Turks, the Greek Byzantine Mathematical School disappeared. Numerous Greeks took refuge in Italy and Europe. Later, they gave considerable stimulus to the study of science in Europe. When in 529 A.D, Justinian closed the pagan philosophical schools in Athens, the schools were dispersed moving to Syria, Persia and elsewhere. They continued to work in Geometry and Arithmetic using the Arabic numeral system.

2.1 Gerbert (940-1003 A.D.) Pope Sylvester II

Gerbert was probably the first who taught Arabic numerals in Europe. He was active in politics, both lay and ecclesiastical. Gebert served in Germany as tutor and advisor to the Roman Emperor, Otto II. Later, in 999, Gebert was elected as the Pope Sylvester II.

2.2 Fibonacci (1170-1250 A.D.)

One of the most talented mathematicians in the thirteen century was Fibonacci (1170-1250 A.D.). Fibonacci traveled along with his father to Egypt, Sicily, Greece and Syria. There, Fibonacci learned methods of calculation using Arabic numerals. Fibonacci published the book Liber Abaci in which he defined the Fibonacci's sequence

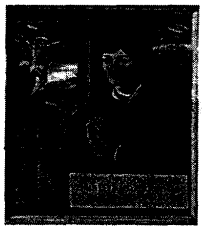
1,1,2,3,5,8,13,21,34,55,.....;

with the general term

$$a_0 = 0,$$
$$a_1 = 1,$$
$$a_{n+2} = a_n + a_{n+1}$$

Many articles have since been published about Fibonacci's sequence. In contemporary times, the sequence plays an important role in the theory of recursive functions and it is extensively used in computer theory and applications.
The fourteen century was relatively unfruitful, mathematically. It was the century of the Black Death, which swept away more than a third of the European population. The Hundred Years War caused social and economical disaster in northern Europe.

2.4 Nicholas Copernicus (1473-1543 A.D.)



Nicholas Copernicus, astronomer and Vicar in the town of Fromborg, Poland, published the book "De Revolutionibus Orbium Celestium Libri Decem" in which he described the model of the solar system with the Sun as the centre. This book contains a substantial section on trigonometry and its applications. There were two outstanding astronomers, Galileo and Kepler, who supported Copernicus' Solar System and had contributed notably to mathematics in the early part of the seventeen century. All three of them were devoted Christians and considered Mathematics as part of the Ideal World. Kepler wrote in his book "Harmony of the World" (1619) that Geometry existed before creation and it is co-eternal with the mind of God.

Galileo Galilei 1564-1642

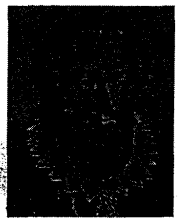
Marie Galileo

In 1623, Galileo wrote: *All knowledge is written in the Great Book the Universe. The Great Book is always opened for us. It is impossible to understand the Great Book not knowing its language which is Mathematics.*
In Physics, Galileo's Principle of Relativity have been extended by Einstein to in the Einstein's Principle of Relativity.
Galileo was devoted Christian and he sent his daughter Marie to a Christian convent

Johannes Kepler 1571-1630

Johannes Kepler published the rigorous mathematical model of the planetary motion in his book Harmony of the World in 1619. In the book Kepler stated three laws of the planetary motion.

Viete (1540-1603 A.D.)



In the sixteen century, Viete was one of the most talented mathematicians. He found the formula

$$x_1 + x_2 = -\frac{b}{a},$$

$$x_1 x_2 = \frac{c}{a},$$

which represents the relation between the roots x_1, x_2 and the coefficients a,b,c of the quadratic equation

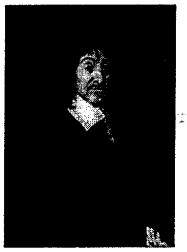
$$ax^2 + bx + c = 0$$

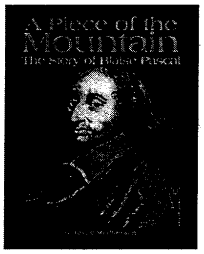
Viete's formulae are included in high schools programs all over the world.

3. Mathematics from the 17th to 20th Century

Let us note that from ancient time until the sixteenth century, Greek Mathematics and Middle Ages Mathematics were about Arithmetic, Euclidean Geometry and Trigonometry.
These subjects of Ancient and Middle Ages Mathematics belong to the high schools programs. Since the seventeen century, new concepts and new subjects within Mathematics have been developed and introduced to the university programs.

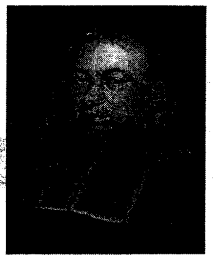
3.1. Descartes (1596-1650 A.D) and Pascal (1623-1662)





Descartes established foundations of Analytic Geometry. Pascal was a contemporary to Descartes.
When Pascal was fourteen years old, he participated in the weekly meetings of a group of French mathematicians. At the age of sixteen, Pascal wrote an essay on conic sections that Descartes could not believe was the work of the boy, assuming that it must be his farther instead. At the age of eighteen he invented the first calculating machine. Using his machine, he assisted his farther in auditing of accounts. Some of Pascal's calculators can still be found in the Conservatory of Art in Paris.

3.2. Fermat (1601-1665).

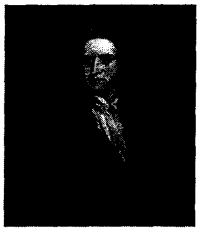


Analytic geometry was also of great interest to Fermat. Fermat claimed that he preceded Descartes in writing the equations of straight line, a circle and a discussion of hyperbola, ellipse and parabola. In literature, Fermat gained popularity from the formulation of Fermat's conjecture "There do not exist natural numbers x, y, z and n such that

$$x^n + y^n = z^n$$

for $n \geq 3$.
Many prominent mathematicians and people of different professions, such as Engineers, Lawyers and Priests, tried to solve the Fermat's conjecture, but the conjecture was open until Wiley's proof published in 1995. Wiley's proof of the conjecture is written in two volumes, the books are about 200 pages each.

3.3. Newton (1642-1727) and Leibniz (1646-1716)



The foundation of calculus, differentiation and integration was established by Newton and Leibniz. These important events have significant impact on further development of mathematics and its different subjects such as Mathematical Analysis, Numerical Analysis, Functional Analysis, Differential Geometry and Topology. Newton devoted most of his life to mathematics and natural philosophy. In social life, he represented the Cambridge University in parliament. He was also the president of the Royal Society.
Leibniz, the great universal genius of the seventeenth century, published papers, independently of Newton, on derivatives and integrals. Also, Leibniz was a particularly gifted linguist, winning some fame as a Sanskrit scholar, and his writings on philosophy have ranked him high in the field.

3.4. Gauss (1777-1855). Euler (1707-1783)

In the eighteenth and nineteenth centuries, Euler and Gauss, among others, were leaders in mathematics and physics. Euler's contributions to mathematics are too numerous to expound completely here. We may only note some of his contributions. First, we owe to Euler the following notations which are used in contemporary time

f(x) the value of the function **f** at the point **x**
a,b,c sides of a triangle

$\sum$ the symbol of a sum

i the imaginary unit $\sqrt{-1}$

Gauss is universally regarded as the greatest mathematician of the eighteenth century and, along with Archimedes and Newton, one of the three greatest mathematicians of all time.
In his doctorate dissertation, written at the age of twenty, Gauss gave the first complete and satisfactory proof of the fundamental theorem of algebra. This proof was unsuccessfully attempted by Newton, Euler, D'Alembert, and Lagrange.

3.5. Lagrange (1736-1813)

Eighteenth and nineteenth centuries were fruitful in the development of new theories in mathematics. Contemporary to Euler, Lagrange's (1736-1813) contribution to mathematics was comparable with Gauss's and Euler's results. Frederic the Great wrote to Lagrange that "the greatest king in Europe" wishes to have at his court "the greatest Mathematician in Europe".
Lagrange's work had a deep influence on the mathematical research. He was the earliest mathematician who recognized the need to introduce rigorous theory of mathematical analysis.

3.6. Fourier (1768-1830) and Cauchy (1789-1857)

Fourier (1768-1830) presented a paper to the French Academy of Science on trigonometric series in which he initiated a new chapter in the history of mathematics. Cauchy (1789-1857), throughout his life remained a devoted Catholic. He developed the theory of complex functions. In the paper on analytic functions, he announced the theorem on representation of complex functions by a power series. In mathematical analysis, he gave a rigorous definition of a limit and a derivative. Cauchy contributed to almost as many fields of study as did Gauss.

3.7 Riemann (1826-1866) and Weierstrass (1815-1897)

Weierstrass (1815-1897) and Riemann (1826-1866) were two outstanding mathematicians of the nineteenth century. Weierstrass was a teacher at a high school up to the age of forty. Later he became a professor at the University of Berlin. In mathematics, Weierstrass achieved many important results. His theorems on continuous functions and on convergence of sequences of functions constitute the foundation of Mathematical Analysis.
Riemann clarified the concept of the integral by the definition of what we now know as the Riemann integral, which led, in the twentieth century, to a more general Lébesque integral. Famous in

mathematical literature are Riemann zeta function and associated Riemann hypothesis. He established the foundation for Riemann geometry by introducing a highly fruitful concept of a Riemann surface.

3.8 Henri Poincare 1854-1912

Henri Poincare made a significant contribution to mathematics and physics. He published about five hundred (500) articles and thirty (30) books. In physics, Poincare stated a preliminary version of the special theory of relativity. Then, the relativity theory was known to a number of mathematicians and physicists before Einstein published his articles. Even the famous Einstein energy formula E=m c^2 was known to Poincare and De Pretto (cf. [15]). Einstein was not only the one who suppose to originate the energy equation E=m c^2 The conversion of matter into energy and energy into matter was much earlier known to Newton.

3.8. Mathematics in Contemporary Times

In contemporary times, mathematics became the main research tool exploring all aspects of human activities. In particular, Mathematics is an important part of programs taught at schools and universities all over the world. Within the last two centuries, new mathematical subjects have been created and cultivated. Among these are: Abstract Algebra, Boolean Algebra, Set Theory and Topology, Mathematical Logic, Game Theory, Modeling, Computing, and earlier Probability and Statistics.
These subjects have significant applications in all aspects of research including Engineering, Sciences, Medicine, Computing and Modeling.
Many new highly sophisticated and interesting professions have been created in the area of Mathematics and related subjects, such as Computing and Modeling. Expectations are continuously growing for new highly qualified professions generated by Mathematics.

Let me finish this lecture with the observation: *All knowledge, discoveries and innovations belong to successive generations, and not to individuals, alone.*

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